Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16
(Autonomous)
Odd Semester Examination: Oct/Nov.2024
Faculty: Science and Technology

Program:BScGen04   **Semester: V**   **SET : A**
Program (Specific): BSc(Mathematics)   **Course Type: core**
Class: TYBSc(Mathematics)   **Max.Marks: 35**
Name of the Course: Ordinary Differential Equations   **Course Code:24-MT-354**
Time: 2Hrs   **Paper: IV**
Instructions to the candidate:

1) *There are 3 sections in the question paper. Write each section on separate page.*
2) *All Sections are compulsory.*
3) *Figures to the right indicate full marks.*
4) *Draw a well labelled diagram wherever necessary.*

SECTION: A

Q1) Answer the following (Attempt any 5 out of 7)   **10 Marks**

1. Verify that $y_1 = \cos x$ and $y_2 = \sin x$ are solutions of $y'' + y = 0$ on $(-\infty, \infty)$.

2. Determine whether $x = 0$ is an ordinary point or a singular point of the differential equation $2x^2 y'' + 7x(x+1)y' - 3y = 0$.

3. Find the value of Wronskian of the functions $y_1 = e^x$ and $y_2 = e^{4x}$.

4. Show that $x = e^{-t}$, $y = -e^{-t}$ are solutions of the homogeneous system

$$\frac{dx}{dt} = x + 2y \ , \ \frac{dy}{dt} = 3x + 2y.$$

5. State the Principle of Superposition.

6. Find the general solution of the differential equation $y'' + 6y' + 5y = 0$.

7. Obtain the particular integral of $(D^2 + 4) y = \sin 3 x$.

SECTION: B

Q2) Answer the following (Attempt any 3 out of 5)   **15 Marks**

1. If y_1 and y_2 are two solutions of $y'' + a_1 y' + a_2 y = 0$ on an interval (a, b) containing a point x_0, then prove that

$$W(y_1, y_2)(x) = e^{-a_1(x-x_0)} W(y_1, y_2)(x_0)$$

2. Obtain the power series solution of differential equation $y'' + x y' + 3y = 0$ about point

$x = 0$.

3. Find the general solution of the differential equation $y'' - 3y' - 4y = 6 e^x$.

4. If $y_1(x) = e^{2x}$ is one solution of the differential equation $y'' - 4y' + 4y = 0$, then find the general solution.

5. Find the general solution of the system $\frac{dx}{dt} = $ x+y , $\frac{dy}{dt} = $ 4x-y.

SECTION: C

Q3) Answer the following (Attempt any 1 out of 2)   **10 Marks**

1. a) By the method of undetermined coefficients, solve $(D^2 - 2D + 1)y = x^2$.

   b) Explain the method of variation of parameters to solve the second order differential equation y"+ P(x) y'+ Q(x) y= R(x).

2. a) Find all solutions of the following differential equations:

   i)$x^2 y'' + xy' - 4y = 0$ for $|x| > 0$.

   ii)$x^2 y'' - 3xy' + 5y = 0$ for $x < 0$.

   b) Prove that: $\dfrac{1}{f(D)} e^{ax} = \dfrac{1}{f(a)} e^{ax}$, when f (a) $\neq 0$.